



# **AISAM-ACME study on non-life long tail liabilities**

**Reserve risk and risk margin assessment under  
Solvency II**

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## Reserve risk and risk margin assessment under Solvency II

### Key conclusions

In the context of Solvency II and the QIS3 exercise, AISAM-ACME, having studied the reserve risk calculation of non-life long tail mutual insurers in 7 countries, found that, based on the QIS3 results and extensive actuarial analyses of a varied sample of 45 supervised insurance companies:

- the Cost of Capital approach is acceptable if the calibration of the reserve risk is adapted,
- the calibration of the reserve risk needs to reflect strictly the one year horizon rather than a full-run off approach. Consequently the use of innovative actuarial methodologies is required to replace the classical ones which are inappropriate.

If the above conditions are fulfilled, participants found that the QIS3 calibration of the reserve risk within the standard formula had to be divided by 2 or 3.

AISAM-ACME believe that the issue of the calibration of insurance risks over a one year horizon may be valid in other areas even if less perceptible.

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## Executive summary

In the context of QIS3, certain members of AISAM and ACME have expressed their concern regarding the high level of the reserve risk calibration within the SCR as designed in QIS3. This is especially the case for non-life long tail insurers with businesses such as workers' compensation and general liability insurance. Upon examination, while the reserve risk calibration is not the only issue for the long tail players, it is the one on which, pragmatically speaking, the Joint Solvency II Working Party of AISAM and ACME believes that a contribution could be useful. Consequently, the secretariats of AISAM and ACME have been commissioned to carry out a study. AISAM and ACME members writing non-life long tail business were invited to work on this issue: In reply 13 mutual insurers, representing 45 supervised companies, from 7 countries, agreed to participate in the study.

From a qualitative point of view, this study aims at clarifying how the reserve risk should be calculated over a one year horizon. The reserve risk should capture unforeseen adverse events occurring over a period of 12 months and the financial consequences of that year's events over the whole run-off of liabilities. This framework is very different from the most common actuarial methodologies which are designed to capture the unforeseen adverse events occurring over the whole run-off of liabilities. To assess the reserve risk over a one year horizon, a distinction must be made between a period of one year over which an adverse event occurs i.e. "shock period" (this adverse event has a probability of 0.5%) and a period over which the adverse event will impact the liabilities i.e. the "effect period". Of course, run-off risks beyond a one year horizon are not outside the scope of Solvency II since they are taken into account in the reserve risk through the "effect period" and they are supposed to be captured in the risk margin specified for technical provisions. We consider that the one year horizon is certainly a "narrow approach" as regards liabilities but it is the same approach as the one which is used to measure risks on assets. In fact, within the Solvency II framework, it should not be a surprise that some long tail business - where adverse movements in claims provisions emerge slowly over many years - requires less solvency capital than some short tail business exposed to catastrophe risks (for instance).

From a quantitative point of view, this study shows that, for the participants in the study, the order of magnitude given for the underwriting risk parameters within QIS3 seems to be consistent with a full run-off approach rather than a one-year-horizon volatility. For participants in this study, the calibration of the reserve risk within the standard formula designed under the QIS3 exercise should be more consistent with the one year horizon if divided by 2 to 3.

In addition, if the reserve risk is over calibrated, the risk margin leads to an inappropriate level of prudence when the cost of capital (CoC) methodology is applied. Indeed, with the calibration of the reserve risk in the QIS3 exercise, the risk margin is more often higher than a value at risk (VaR) at 90%. This collateral effect creates a double penalization. Therefore, the CoC methodology seems to be acceptable only if the reserve risk is calibrated strictly over a one year horizon. Under that condition (namely the calibration of the reserve risk over a one year horizon), the cost of capital provides a VaR which becomes acceptable in most cases.

Although the solution of using an internal model to address the assessment of the solvency capital required for specific lines of business (like long tail business) is acceptable, this can only be the case if the benchmark, i.e. the standard formula, is not too far from the “real risk”. Indeed, the larger the gap between the standard formula and the internal model, the more difficult the validation of results from the internal model by the supervisors may be. Therefore the calibration of the standard formula remains of crucial importance for all long tail players.

Finally, we believe that the issue of the calibration of insurance risks over a one year horizon may be valid in other areas even if it is less perceptible. In non-life lines, as we demonstrate in the study, the reserve risk of each line of business is less important for a one year horizon than for classical assessments on the ultimate costs even for short tail business. Broadly speaking we consider that the premium risk (in non-life) and the shocks on the life risk modules (longevity, mortality, disability and revision) should also be discussed and checked by the insurance industry, regarding the framework of Solvency II.

## **AISAM & ACME Presentation**

The Association Internationale des Sociétés d'Assurance Mutuelle/International Association of Mutual Insurance Companies (AISAM) and the Association of European Cooperative and Mutual Insurers (ACME) represent about 80% of the mutual insurance market in Europe and over 6% of the worldwide insurance premiums. Its members are present in at least 16 European countries and employ over 300,000 people either directly or through their subsidiary companies.

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# 1 Why this study?

## 1.1 Calibration of the standard formula under the QIS3 exercise

Some members have expressed their concern regarding the high level of the reserve risk component within the SCR as designed in QIS3 for non-life long tail business such as workers' compensation insurance (line of business n°1; volatility in QIS3 = 15%) and general liability insurance (line of business n°8; volatility in QIS3 = 15%). This order of magnitude given for the underwriting risk parameters within QIS3 seems to be consistent with a full run-off approach rather than a one year horizon volatility. This difference between the two time horizons is all the more important and perceptible the longer the duration. It is therefore for these reasons that the study is focused on these lines of business (average duration of 5 to 6 years).

The fact that the current assessments rely on the UK market only and the lack of information on how CEIOPS has made the assessment<sup>1</sup> of the reserve risk have contributed to our decision to launch this study.

In addition, the CoC method which mainly relies on the reserve risk (for non-life long tail players) "mechanically" provides penalizing results. This collateral effect creates a double penalization.

## 1.2 The common practice within member companies

AISAM-ACME have questioned some mutual insurer members, who are developing stochastic assessments of their provisions, as to how they calculate the standard deviation of their provisions. Almost all of them assess the standard deviation of their reserves on the ultimate costs because:

- The traditional use of stochastic methodology is to value the distribution of reserves to measure their level of prudence in the balance sheets: of course this measurement relies on full run-off approach,
- To address this need for valuation, the most common actuarial approaches are used which are based on a full run-off approach.

Only a few members were aware of the inconsistency between their assessment on the ultimate costs and the Solvency II framework which uses a one year horizon. And yet the one year horizon is of crucial importance to calibrate properly the reserve risk, especially for long tail liabilities.

## 1.3 A need to clarify the reserve risk calibration

Given this context, the AISAM-ACME Solvency II Working Party has decided to propose to member companies which write non-life long tail business to provide a specific contribution on this issue by:

- Disclosing the relevant QIS3 results (we agreed with participants that the disclosure of results will remain anonymous),
- Providing entity specific assessments,

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<sup>1</sup> See the calibration paper from CEIOPS at the following address (page 15): <http://www.ceiops.org/media/files/consultations/QIS/QIS3/QIS3CalibrationPapers.pdf>

- Developing technical clarifications on this issue.

## 1.4 The calibration of the Standard Formula remains the benchmark

The solution whereby an internal model is used to address the assessment of the solvency capital required by specific lines of business (like long tail business) is perfectly acceptable provided that the benchmark, i.e. the standard formula, is not too far from the “real risk”. Indeed, the greater the gap between the standard formula and the internal model, the more difficult the validation of results from the internal model may be.

## 2 Design of the study

### 2.1 Contents

This study consists of:

- A description of the participants in this study and their relevant QIS3 results. These contributions are anonymous,
- A development of the concept of a one year horizon applied to the reserve risks,
- Comparative results between entity specific assessments and the QIS3 calibration,
- A specific development on the cost of capital approach.

### 2.2 Organization

This study was carried out from May 2007 to October 2007. AISAM and ACME secretariats have gathered and structured the results provided by the participants in the study. Almost all the assessments have been carried out by the participants in the study since modelling long tail business requires solid experience and an extensive knowledge of the underlying risks.



## 3 Description of participants and QIS3 results

### 3.1 Profile of participants

This study includes 13 participants from 7 countries (Table 1) with a majority of specialized players<sup>2</sup> (Table 2). These 13 participants represent 45 supervised companies, of which 20 mutual insurers (one participant can be a group of supervised entities or a solo entity). The majority of participants write general liability (Table 3). There is a wide range of size of participants (Table 4) but, since these players write long tail business, the size of technical provisions is quite high (Table 5): on average, technical provisions represent 415% of the gross written premium.

Table 1: participation per country

Belgium	3
Germany	1
Denmark	1
Spain	2
Finland	1
France	4
Sweden	1

Table 2: participation per level of diversification

Specialized	9
Diversified	4

Table 3: participation per line of business

Workers' Compensation	2
General Liabilities	8
Both	3

Table 4 : participation per gross written premium

EUR m	
< 100	3
100 - 500	6
> 500	4

<sup>2</sup> To define a specialized player, we have chosen the following criteria: an insurer is specialized in a line of business if this line of business represents more than a half of its written premium or more than a half of its technical liabilities. The diversified players represent the insurers which do not correspond to the definition of a specialized insurer.

Table 5 : participation per level of technical provisions

EUR m	
< 500	3
500 - 2000	9
> 2000	1

### 3.2 QIS3 results

Even if a solvency position under Solvency I can not be strictly compared to the solvency position under QIS3 (two thresholds), it is clear that the solvency position of participants under QIS3 has greatly deteriorated (Table 6). The situation is particularly worrying for specialized players since 6 out of 9 breach the SCR. For two of the mutual insurers which breach the SCR, their ratings do not mirror the QIS3 results.

Table 6 : Solvency position under Solvency I vs. QIS3 (SCR) – all participants

	average	min	max
Solvency I	495%	209%	1625%
QIS3 (SCR)	129%	28%	294%

Table 7 : Solvency position under Solvency I vs. QIS3 (SCR) – specialized players

	average	min	max
Solvency I	356%	211%	719%
QIS3 (SCR)	105%	28%	182%

As we will demonstrate below, the QIS3 reserve risk calibration and the cost of capital are among the most important reasons that explain this situation.

## 4 The concept of the one year horizon for the reserve risk

The uncertainty measurement of reserves in the balance sheet (called risk margin in the Solvency II framework) and the reserve risk do not have the same time horizon. It seems important to underline this point because it may be a source of confusion when the calibration is discussed.

### 4.1 Comparison between the reserve risk and the risk margin

#### 4.1.1 The risk margin captures uncertainty over the whole run-off of liabilities

##### 4.1.1.1 The Solvency II draft Directive framework

The recast draft directive provides a definition of the risk margin (P92-93):

*“3. The risk margin ensures that the overall value of the technical provisions is equivalent to the amount (re)insurance undertakings would expect to have to pay today if it transferred its contractual rights and obligations immediately to another undertakings; or alternatively, the additional cost, above the best estimate of providing capital to support the (re)insurance obligations over the lifetime of the portfolio”*

For non-life liabilities (which are non-hedgeable in general) the risk margin is the financial cost of uncertainty of liabilities over the whole run-off giving that this uncertainty is calibrated through the solvency filter:

*“5. Where insurance and reinsurance undertakings value the best estimate and the risk margin separately, the risk margin shall be calculated by determining the cost of providing an amount of eligible own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the lifetime thereof.”*

##### 4.1.1.2 Tools to measure the risk margin

The cost of capital (CoC) and the value at risk (VaR) methods are the main tools to calculate a risk margin. These two methodologies have a common point which is to capture the uncertainty over the whole run-off of the reserves:

- The CoC relies on a projection of the Solvency Capital Required (wording in the Solvency II framework, see in the next paragraph the definition of the Solvency Capital Required) to face potential adverse events until the last payment of liabilities, i.e. over the whole run-off of the reserves,
- The VaR relies on a probability distribution of the projected ultimate costs.

## 4.1.2 The reserve risk captures uncertainty over a one year period

### 4.1.2.1 The Solvency II draft Directive framework

The SCR has the following definition<sup>3</sup>:

*“The SCR corresponds to the economic capital a (re)insurance undertaking needs to hold in order to limit the probability of ruin to 0.5%, i.e. ruin would occur once every 200 years (see Article 100). The SCR is calculated using Value-at-Risk techniques, either in accordance with the standard formula, or using an internal model: all potential losses, including adverse revaluation of assets and liabilities over the next 12 months are to be assessed. The SCR reflects the true risk profile of the undertaking, taking account of all quantifiable risks, as well as the net impact of risk mitigation techniques.”*

Within this framework, the reserve risk is defined as a part of the underwriting risk<sup>4</sup>, as follows:

*“(24) Underwriting risk means the risk of loss, or of adverse change in the value of insurance liabilities, due to inadequate pricing and provisioning”*

### 4.1.2.2 Consequences of this framework on the reserve risk assessment

The following paragraphs refer to the IAIS<sup>5</sup> Guidance paper on the structure of regulatory capital requirements (draft version)<sup>6</sup>.

#### “A shock period and an effect period

If we apply this framework to the reserve risk, the concept of time horizon should distinguish between:

- a period of one year over which an adverse event occurs i.e. “shock period”; this adverse event has a probability of 0.5% and,
- a period over which the adverse event will impact the liabilities i.e. the “effect period”.

In any case the reserve risk should capture the risks arising over the occurrence period and their financial consequences over the whole run-off of liabilities.

- For example, a court judgement or judicial opinion in one year (the shock period) may have permanent consequences for the value of claims and hence will change the projected cash flows to be considered over the full run-off of liabilities (the effect period).

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<sup>3</sup> Recast directive, page 11

<sup>4</sup> Underwriting risk covers the premium risk and the reserve risk

<sup>5</sup> International Association of Insurance Supervisors

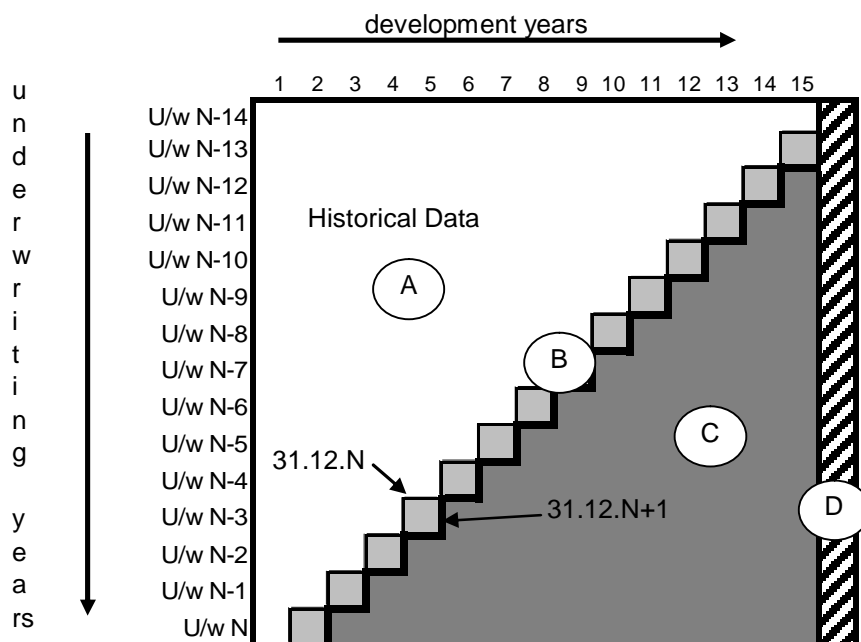
<sup>6</sup> The final version of this paper will be available by the end of 2008 at the following address: <http://www.iaisweb.org>

### Illustration

To illustrate the concept of a one year horizon year, let's consider the following example. The goal is to assess the reserve risk at 31.12.N over a one year horizon, from the triangulation of losses<sup>7</sup> over 15 underwriting years [U/w N-14;U/w N].

The following picture is divided into 4 areas (A, B, C, D)

Picture 1



**Area A:** This area contains the available data/information at 31.12.N to assess the reserves at 31.12.N (Noted  $R_n$ ).

**Area B:** This area (soft grey) corresponds to a one year period beyond 31.12.N. This area represents the “shock period”. At the end of the shock period (i.e. at 31.12.N+1), it will be possible to revise  $R_n$  *a posteriori* considering:

- The real payments of losses (noted  $P_n$ ) over the period [01.01.N+1; 31.12.N+1],
- The valuation of reserves at 31.12.N+1 (noted  $R_{n+1}$ ) regarding the available information until 31.12.N+1 for the underwriting years [U/w N-14;U/w N]

The reserve risk at 31.12.N measures the uncertainty of the valuation of reserves calculated at 31.12.N regarding the additional information over the period [01.01.N+1; 31.12.N+1] that could change this valuation at 31.12.N+1<sup>8</sup>. The reserve risk captures the difference between  $[P_{n+1} + R_{n+1}]$  and  $R_n$ .

<sup>7</sup> For this illustration:

- the triangulation can be developed on cumulative (or not) payments or total charge...
- the underwriting year can be changed into an occurrence basis.

<sup>8</sup> The reserves at 31.12.N+1 do not include the liabilities related to the underwriting year N+1. Indeed the risk associated with this underwriting year is captured in the premium risk

**Area C:** Under the Solvency 2 framework and to calculate the reserve risk, this area represents the effect period beyond the shock period. This area contains additional information that could lead to revision of the reserves beyond 31.13.N+1. This additional information should not be taken into account. The use of the area C should be limited to the assessment of the financial consequences of the adverse events arising during the shock period.

**Area D:** this area contains the ultimate costs. These costs are used to assess the risk margin with a VaR methodology. The most usual actuarial methodologies (Mack Model, Bornhuetter-Ferguson, Bootstrapping methodologies on incremental payments) are not consistent with the Solvency II framework since they capture all the adverse events arising beyond the one year horizon.

#### **4.1.2.3 General comments**

Within the Solvency II framework, it should not be a surprise that some long tail business where adverse movements in claims provisions emerge slowly over many years require less solvency capital than some short tail business exposed to catastrophe risks (for instance).

## 4.2 Technical solutions

The technical solutions, which are briefly described in the following paragraphs, rely on the scientific articles that participants have used to develop their tools to assess the reserve risk. Overall, we have found very few references on this topic. Therefore, we have decided additionally to include a very simple approach based on the use of retrospective accounting data.

### 4.2.1 Assessment based on historical accounting data

Following discussions with several participants in this AISAM-ACME study, it became clear that we could include in the study a very simple measurement of the reserves over a one year horizon. This approach relies on historical accounting data and it has two advantages: firstly it's simple, and secondly it enables many participants in the study to provide an assessment of the historical volatility of their reserves.

#### "Methodology"

Let's consider the amount of reserves (noted  $R_n$ , including outstanding claims and IBNR) which has been booked in the balance sheet at 31.12.N for all the underwriting years prior to N and N. It is possible to see what the deviation at 31.12.N+1 has been for these underwriting years (i.e. excluding the year N+1), considering:

- The payments which have been made during the year N+1 ( $P_n$ ),
- The new evaluation of the reserves at 31.12.N+1 ( $R_{n+1}$ ).

The historical volatility is calculated as follows:  $\frac{R_n - (R_{n+1} + P_n)}{R_n}$  (%).

#### Comments

Of course this valuation is not prospective; however, this assessment can be helpful in providing an order of magnitude regarding the volatility of reserves (less than 5%, 5-10%, 10%-15%, more than 15%...).

#### 4.2.2 Stochastic reserving models compatible with a one year horizon

In the following paragraph, we give a short description of the stochastic methodologies that participants have developed:

- The scientific references which have been used by participants (see appendix 1),
- A description of how each participant has carried out his/her assessment of the company's business (see appendix 2).

Indeed, assessing reserve risks requires a customization of methodologies for each business.

##### 4.2.2.1 *The scientific references*

Participants have used 3 kinds of methodologies (see appendix 1, page 25):

- An adaptation of the Mack method,
- A Bayesian approach,
- A methodology derived from the Mack and Bootstrapping methods.

##### 4.2.2.2 *The inflation modelling*

A key issue in reserving is the assessment of future inflation (monetary inflation, and/or inflation due to the evolution of the judicial opinion for instance). This question is relevant both for the best estimate of liabilities and for the reserve risk. Nevertheless, this issue must be addressed differently from the best estimate of liabilities point of view and the reserve risk point of view.

Indeed:

- Concerning the best estimate of liabilities, the key issue is the choice of the average inflation over the whole run-off (i.e. the trend).
- Concerning the reserve risk the key issue is the risk in changing the trend chosen for the best estimate of liabilities with the additional information of the next 12 months. The question relies on the volatility of the trend and not the trend itself.

To address the inflation risk, within the assessment of the prediction errors, two approaches are used.

The first approach (and the most used) relies on the replication of the past volatility of inflation. In general, participants have found neither a relevant retrospective deflator nor a relevant prospective inflator. The participants have therefore decided to replicate the past volatility of inflation in the future, all the more so given that the line of business underwritten has faced acute changes in the inflation trend in the past and/or significant shocks in the patterns of payments (because of adverse court cases for instance). To that extent the use of historical data without changes appears to be cautious. The relevance of the use of the past intrinsic inflation to make a projection in the future was discussed and remains



certainly a pending issue. However, in this study, participants who have used historical costs have not identified a real need for introducing, for instance, an additional volatility over the following 12 months (the volatility of inflation already exists in the historical triangles).

The second approach relies on the deflation of historical costs and then a stochastic inflation of the prospective costs. Only one participant developed this approach because he/she succeeded in finding a relevant deflator for historical costs and because he/she wanted an explicit identification of the inflation risk. This approach raises the question of calibration of the inflation risk over a one year horizon. It seems that autoregressive models, applied to the trends, could address this issue. Nevertheless it must be analysed through the one year horizon view, which dramatically reduces the order of magnitude of the potential shift in the inflation trend.

#### *4.2.2.3 Description of the models used by each participant*

This study presents in a harmonized way the models which have been used by participants. Each participant describes (see appendix 2, page 25):

- His/her experience in modelling reserve risk, and its use in the day-to-day operations of the company,
- The data history: extent of historical data, the treatment of historical data if any (deflation, treatment of large claims...),
- The data processing in a prospective mode both for the full run-off approach and for the one year horizon: which models have been tested, which ones have been used and why, what is the treatment of inflation (monetary inflation and inflation due to court actions if relevant)...?,
- The areas for improvement.

## 4.3 Comparative results between entity specific data and QIS3 formula

### 4.3.1 Results from treatment of historical accounting data

This simple methodology aims only at providing an order of magnitude regarding the volatility of reserves (less than 5%, 5-10%, 10%-15%, more than 15%...). Accurate assessments are made through prospective stochastic methodologies, the results of which are disclosed in the following paragraphs.

Many participants have faced difficulties in gathering net of reinsurance data. These results therefore rely on gross of reinsurance reserves and provide an over calibration of the risk we are trying to measure. The retrospective standard deviation for each participant is in general lower than 5% (see Table 9). Participants n°11 & n°12 have faced a high (gross of reinsurance) volatility of their reserves (13.1% and 11.8% respectively), mainly because of the very small size of their portfolio (Best estimate of liabilities lower than EUR 60 million). However it is important to note that the non-proportional reinsurance protection of their portfolios enables them to decrease significantly their volatility<sup>9</sup>. Participant n°5 has seen a high volatility in his/her reserves over 3 years (out of 10 years). This period corresponded to the development of a new partnership in the workers' compensation line of business which made it difficult to effectively judge the appropriate level of reserves. Thanks to the experience acquired over this adverse period, the participant has learnt to master the reserving process properly. The prospective assessment of the reserve risk (see the stochastic assessments made by this participant) is therefore significantly lower than the retrospective results.

Picture 2 below shows that the retrospective revisions of reserves have a coherent structure per participant (for each participant, these revisions are grouped around an average revision); the average standard deviation per participant is 5.5% (see Table 9). This could be interpreted as a specificity or habit of each company in their reserving. We remove this "company effect" by calculating an overall standard deviation; the standard deviation (gross of reinsurance) of the reserves revisions is 9.1%<sup>10</sup> (see Table 9).

Therefore, according to this retrospective approach, and for the participants in this study, the order of magnitude of the QIS3 calibration should be between 5% and 10%.

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<sup>9</sup> Participant n°11: the net of reinsurance data are available over the 6 last accounting years. The net of reinsurance volatility is 5.1%. For participant n°12 the volatility over the 10 last accounting years is 9.5%.

<sup>10</sup> This overall standard deviation represents the volatility of the whole set of results given by the participants, regardless of the structure of results by participant previously identified

**Table 8 : Standard deviation of the reserve risk over a one year horizon with a re-treatment of accounting data**

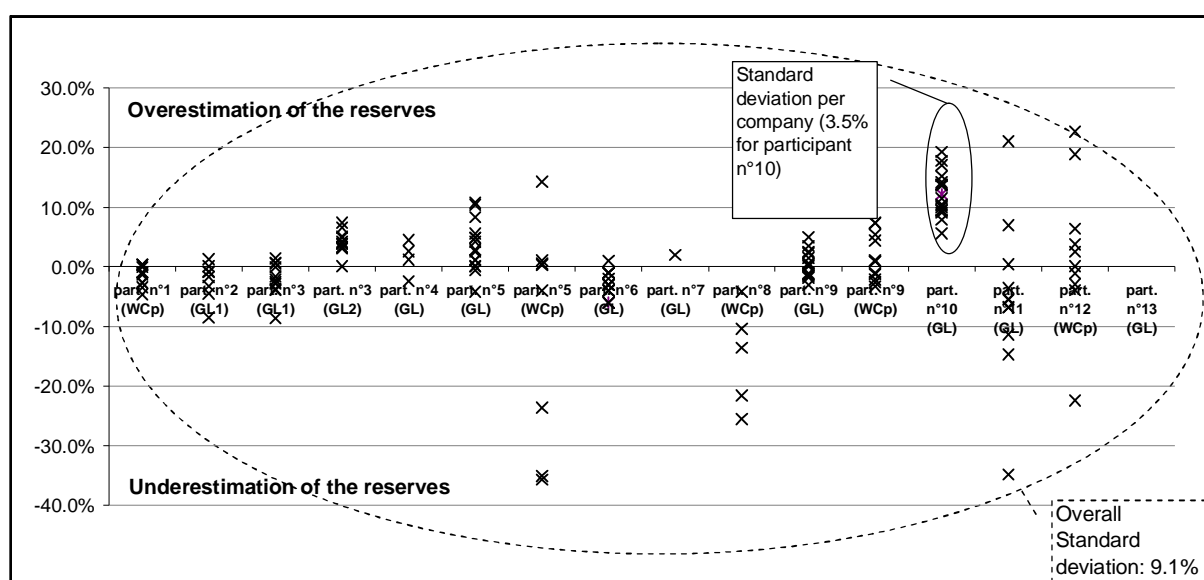
	Standard deviation of accounting reserves (gross of reinsurance)	Number of accounting years used in the calculation
part. n°1 (WCp1&2)	1.6%	11
part. n°2 (GL1)	2.9%	8
part. n°3 (GL1)	2.7%	11
part. n°3 (GL2)	1.8%	11
part. n°4 (GL)	2.5%	4
part. n°5 (GL)	4.0%	15
part. n°5 (WCp)	17.5%	8
part. n°6 (GL)	2.0%	16
part. n°7 (GL)	N/A	N/A
part. n°8 (WCp)	7.7%	5
part. n°9 (GL)	2.2%	13
part. n°9 (WCp)	3.6%	12
part. n°10 (GL)	3.5%	18
part. n°11 (GL)	13.1%	11
part. n°12 (WCp)	11.8%	10
part. n°13 (GL)	N/A	N/A
Average Standard deviation (gross of reinsurance)	5.5%	
Overall Standard deviation (gross of reinsurance)	9.1%	
<b>QIS3 calibration (net of reinsurance)</b>	<b>15.0%</b>	

N/A: the participant has not made the calculations.

GL1 and GL2: 2 lines of business within the GL category are assessed separately

WCp1&2: 2 lines of business within the WCp category are merged for this assessment

**Picture 2**



### 4.3.2 Results from stochastic models

We have included the gross and net assessment of reinsurance, over a one year horizon and over the whole run-off. Where participants have used several methodologies, Table 9 discloses a range of results. Since a majority of participants have not made the calculation net of reinsurance (complexity of treatment of the non-proportional reinsurance, problem to collect data...), the volatilities net of reinsurance are not assessed directly but are presented as lower than the volatilities gross of reinsurance.

Table 9 shows that the QIS3 calibration (15%) is consistent with the volatility assessed by participants over the whole run-off. According to the assessments from participants, the calibration of the reserve risk should be divided by 2 to 3 (see Table 10).

Table 9

		Gross of reinsurance	Net of reinsurance	
<b>over the WHOLE RUN -OFF</b>	participant n°1 (WCp1)	5.10%	<5.10%	
	participant n°1 (WCp2)	2.07%	<2.07	
	participant n°2 (GL1)	9.6%-10.3%	<10.3%	
	participant n°2 (GL2)	9.5%-11.2%	<11.2%	
	participant n°3 (GL)	7.7%	<7.7%	
	participant n°5 (GL)	10.6%-15.6%	<15.6%	
	participant n°5 (WCp)	8.8%-9.5%	<9.5%	
	participant n°9 (GL)	13.5%	<13.5%	
	participant n°10 (GL)	6.2%	4.7%	
	<b>over a ONE YEAR horizon</b>	participant n°1 (WCp1)	4.70%	
participant n°1 (WCp2)		1.79%	< 1.8%	
participant n°2 (GL1)		5.5%-7.7%	< 7.7%	
participant n°2 (GL2)		5.6%-7.6%	< 7.6%	
participant n°3 (GL)		3.8%	< 3.8%	
participant n°5 (GL)		5.4%-5.9%	< 5.9%	
participant n°5 (WCp)		6.3%-6.4%	< 6.4%	
participant n°9 (GL)		8.2%	< 8.2%	
participant n°10 (GL)		4.9%	< 4.9%	
			<b>QIS3 calibration: 15%</b>	

WCp: Workers' Compensation, line of business n°1 under QIS3

GL: General Liabilities, line of business n°8, under QIS3

GL1 and GL2: 2 lines of business within the GL category are assessed separately

WCp1 and WCp2: 2 lines of business within the WCp category are assessed separately

Table 10 : over estimation of QIS3 calibration for the participants in the study

	Over estimation of the QIS3 calibration
participant n°1 (WCp1)	more than 219%
participant n°1 (WCp2)	more than 738%
participant n°2 (GL1)	more than 95%
participant n°2 (GL2)	more than 97%
participant n°3 (GL)	more than 295%
participant n°5 (GL)	more than 154%
participant n°5 (WCp)	more than 134%
participant n°9 (GL)	more than 83%
participant n°10 (GL)	more than 206%

### 4.3.3 Process error and estimation error

We have split the standard deviation (prediction error) into two parts: the process error and the estimation error:

- The process error is the pure stochastic error due to the volatility of claims,
- The estimation error is the parameter or the model error, since in any stochastic model, we need to estimate parameters.

Table 11 below shows that the two components of the prediction error both decrease significantly according to the time horizon. That means, in particular, that the risk in revising the model parameters is significantly lower when it is assessed over one year, compared to the risk over the whole run-off.

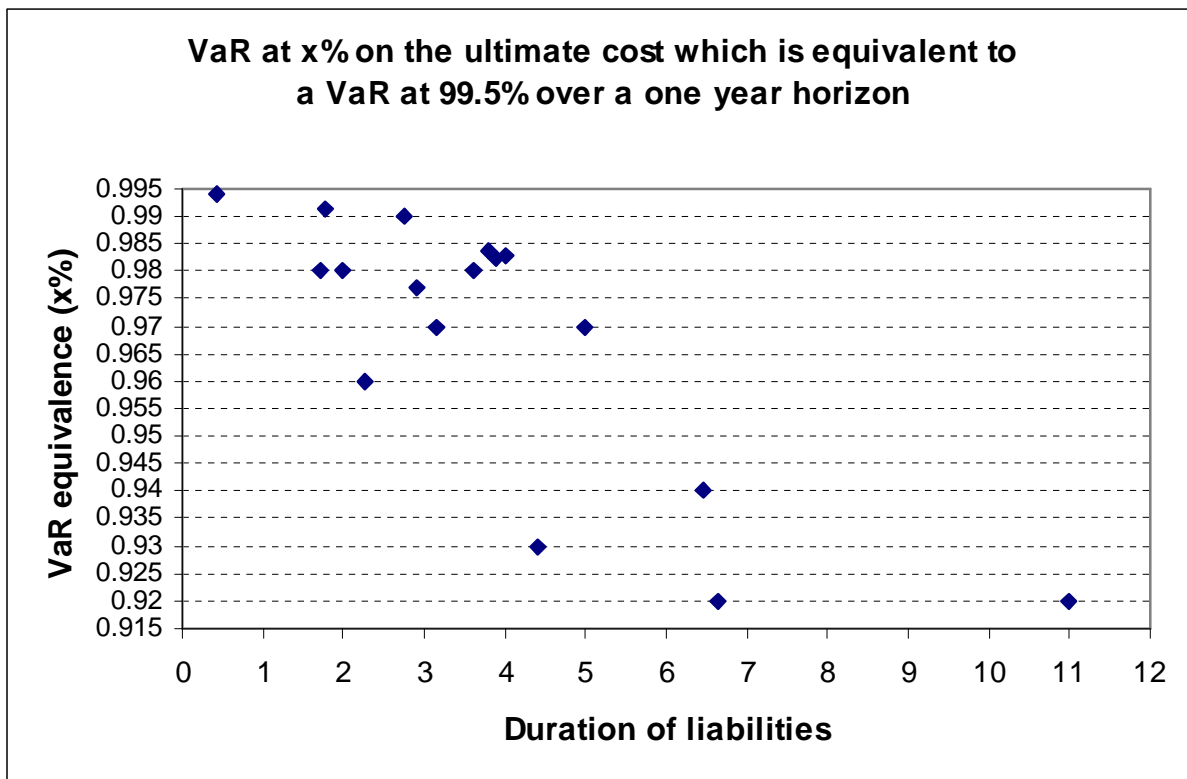
Table 11

	Process error (intrinsic volatility)			Estimation error (model error)			Prediction error (total)		
	Whole run-off	One year horizon	Variation (%)	Whole run-off	One year horizon	Variation (%)	Whole run-off	One year horizon	Variation (%)
participant n°1 (WCp1)	4.60%	4.34%	-6%	2.10%	1.81%	-14%	5.10%	4.70%	-8%
participant n°1 (WCp2)	1.48%	1.23%	-17%	1.45%	1.30%	-10%	2.07%	1.79%	-14%
participant n°2 (GL1)	4.40%	1.90%	-57%	6.60%	3.00%	-55%	7.90%	3.60%	-54%
participant n°2 (GL2)	4.80%	2.50%	-48%	6.80%	3.20%	-53%	8.30%	4.10%	-51%
participant n°3 (GL)	4.65%	2.54%	-45%	6.15%	2.80%	-54%	7.70%	3.78%	-51%
participant n°5 (GL)	5.23%	2.03%	-61%	9.19%	4.96%	-46%	10.58%	5.36%	-49%
participant n°5 (WCp)	6.91%	5.56%	-20%	5.51%	3.42%	-38%	8.84%	6.53%	-26%
participant n°9 (GL)	6.80%	4.80%	-29%	11.60%	6.60%	-43%	13.50%	8.20%	-39%
participant n°10 (GL)	5.05%	3.77%	-25%	3.62%	3.17%	-12%	6.21%	4.93%	-21%

#### 4.4 Alternative methodology: the VaR equivalence

An alternative approach consists of relying on the current methodologies based on a run-off approach. Instead of redefining the modelling approaches on the reserve risk to make them consistent with a one year horizon, an equivalence could be found between a VaR at 99.5% over a one year horizon and a VaR at  $x\%$  over a full run-off according to the duration of liabilities. For a short term business,  $x$  would be close to 99.5% and for long tail business;  $x$  would be significantly lower than 99.5%. Picture 3 shows that the longer the duration the lower the VaR equivalence<sup>11</sup>.

Picture 3



<sup>11</sup> In other words: The VaR equivalence represents the probability of the occurrence of an extreme event over the whole run-off of liabilities with a magnitude which is equal to the VaR at 99.5% over a one-year period.

Let's consider:

- L: the Loss distribution over the full run-off
- VaR99.5: the VaR over a one-year period at a confidence level of 99.5%

The VaR equivalence  $x$  is defined as:  $P(L < \text{VaR}_{99.5}) = x$

This analysis relies on different lines of business. Table 12 (below) presents the number of contributions per line of business (the same as for the QIS3 exercise analyzed).

Table 12

Accident and health - workers compensation	4
Accident and health - health insurance	1
Accident and health - others/default	1
Motor, third party liability	1
Motor, other classes	1
Marine, aviation and transport	1
Fire and other damage to property	1
Third-party liability	4
Credit and suretyship	0
Legal expenses	0
Assistance	0
Miscellaneous non-life insurance	0
<b>TOTAL</b>	<b>14</b>

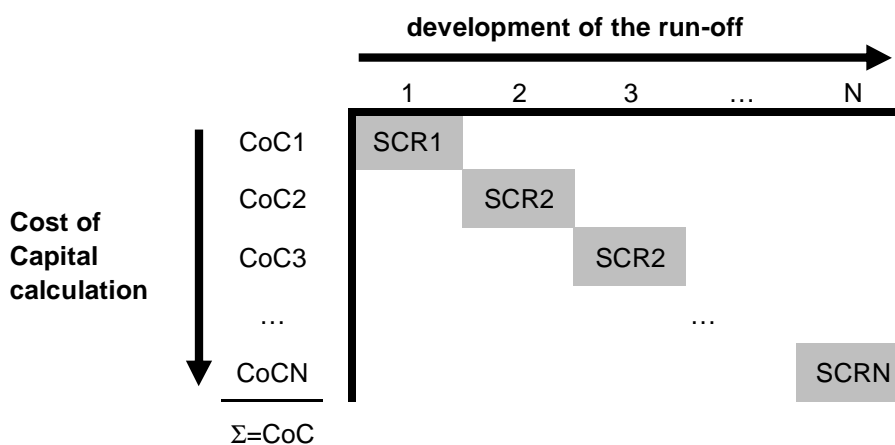
## 5 The Risk margin assessed with the cost of capital methodology

### 5.1 Link between the reserve risk and the cost of capital methodology.

The level of the CoC relies essentially on the reserve risk calibration<sup>12</sup>. If the reserve risk is over calibrated (i.e. for instance a calibration over the whole run-off of the reserves), the CoC methodology multiplies the level of prudence.

Picture 4

Cost of capital calculation if the reserve risk is assessed over a one year horizon:



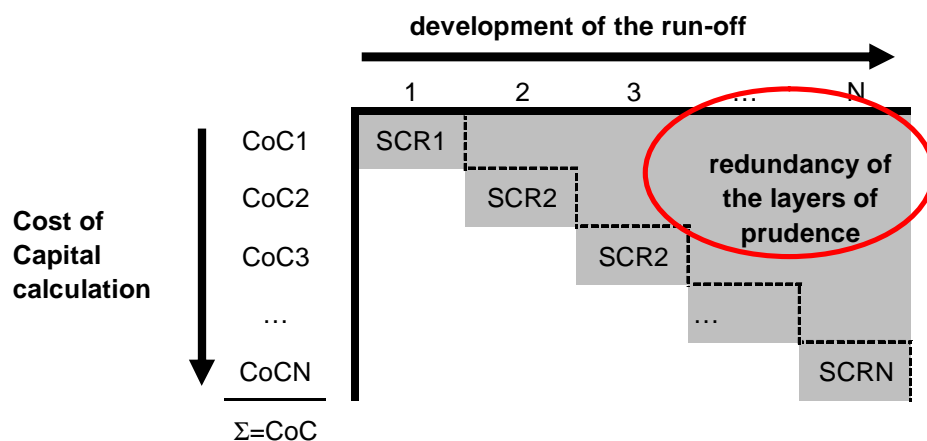
For each year horizon, the CoC captures the cost of providing own funds equal to the Solvency Capital Requirement necessary to support the insurance and reinsurance obligations over the run-off. If the duration of the run-off is N years, the CoC embeds N SCR valuations.

<sup>12</sup> See the QIS3 technical specifications from CEIOPS



Picture 5

Cost of Capital calculation if the reserve risk is assessed over the whole run-off:



If the reserve risk is calibrated over the whole run-off, or, broadly speaking, if the reserve risk is over calibrated, the CoC creates undue layers of prudence with a leverage effect (see the N “clusters of risks” in Picture 4 versus  $N(N+1)/2$  “clusters of risks” in Picture 5).

## 5.2 Results regarding the Cost of Capital

To illustrate this issue (Table 13), participants have calculated the CoC using both:

- the QIS3 calibration of the reserve risk,
- their calibration of the reserve risk over a one year horizon.

The result of this calculation has been translated into:

- a percentage of the best estimate of liabilities (%BEL),
- a VaR at  $x\%$  so that  $x$  provides the same amount as the CoC (VaR equivalence).

When using the QIS3 calibration of the reserve risk, the CoC is equivalent to a VaR which in general is above 90% and even sometimes above 95%. Even if the CoC methodology provides a VaR which is higher for long tail business than for short tail business<sup>13</sup>, we consider that the level of prudence due to the reserve risk calibration is unacceptable. For the record, the first quantitative impact study fixed a risk margin with a VaR at 75%.

We consider that the reserve risk calibration and not the CoC methodology is the cause of the problem for long tail players. Indeed, when the participants calibrate the reserve risk with volatility over a one year horizon the CoC methodology provides a level of prudence which seems to be reasonable (VaR between 70% and 85% in most cases).

<sup>13</sup>See the paper “Exposure Draft on the Measurement of Liabilities for Insurance Contracts” (pages 67-68) issued by the International Actuarial Association (IAA) available at the following address: [http://www.actuaries.org/CTTEES\\_RISKMARGIN/Documents/RMWG\\_Exposure\\_Draft.pdf](http://www.actuaries.org/CTTEES_RISKMARGIN/Documents/RMWG_Exposure_Draft.pdf)

Table 13

	% BEL	VaR equivalence	% BEL	VaR equivalence
participant n°1 (WCp1)	14%	99.9%	2%	88%
participant n°1 (WCp2)	4%	78.9%	2%	64%
participant n°2 (GL1)	12%	85.5%	7%	71.5%
participant n°2 (GL2)	12%	93.0%	7%	79.3%
participant n°3 (GL)	13%	97.5%	4%	75%
participant n°4 (GL)	22%	N/A	N/A	N/A
participant n°5 (GL)	27%	99.2%	11%-12%	82.5%
participant n°5 (WCp)	14%	94.6%	7%	80.9%
participant n°6 (GL)	13%	N/A	N/A	N/A
participant n°7 (GL)	19%	N/A	N/A	N/A
participant n°8 (GL)	13%	N/A	N/A	N/A
participant n°8 (WCp)	21%	N/A	N/A	N/A
participant n°9 (GL)	20%	90%-95%	13%	78.0%
participant n°10 (GL)	15%	99.8%	8%	93.9%
participant n°11 (GL)	18%	N/A	N/A	N/A
participant n°12 (WCp)	5%	N/A	N/A	N/A
participant n°13 (GL)	13%	N/A	N/A	N/A

*N/A: the participant has not made the calculations.*

### 5.3 Proposals

For the participants in the study a proper calibration of the volatility factor in the reserve risk is a key condition to obtaining an acceptable risk margin when it is computed with the CoC methodology. Therefore, the first proposal is the revision of the reserve risk calibration.

For the participants in the study, if the reserve risk remains at the current calibration (15% for general liabilities and workers' compensation), the CoC methodology provides an unacceptable level of prudence. In that case our proposal would be to calculate the risk margin with a VaR.

## Appendix 1: Scientific references

This part of the report is not an exhaustive review of the existing methodologies which can address the assessment of the reserve risk over a one year horizon. We have included here the methodologies which have been used by the participants in the study. The articles which are briefly described below provide a wider view of the existing actuarial developments on this topic. Where necessary, we have provided additional technical developments which aim at adapting the methodologies to the purpose of the study.

### A/ The adaptation of the Mack model (Wüthrich, Merz, Lysenko)<sup>14</sup>

The work of Michael Merz, Mario V. Wüthrich and Natalia Lysenko was initially motivated by the Swiss Solvency Test whose time horizon is one year (see the first article<sup>15</sup> published in 2006 by Michael Merz and Mario V. Wüthrich which is available at the following address: <http://www.math.ethz.ch/~wueth/Papers/AbwErg2.pdf>).

A second (more detailed) article<sup>16</sup> was published in 2007 and is available at the following address: [http://www.math.ethz.ch/~wueth/Papers/CDR\\_WuMeLy.pdf](http://www.math.ethz.ch/~wueth/Papers/CDR_WuMeLy.pdf)

These articles give a mathematical approach for the estimation of the conditional mean square error of prediction of the expected claims development result for the next accounting year. The formula is based on the Chain ladder method and its time series formulation in Buchwalder et al<sup>14</sup>. These articles also provide a split in the prediction error between the process error and the estimation error.

### B/ The Bayesian Methodology (Scollnik)

B.1/ The general concept of the Bayesian Approach is the following:

*Probabilities are conditional*

Probabilities specify the degree of our belief in some proposition(s) under the assumption that some other propositions are true. We require the conditioning propositions to include, at least implicitly, all of the information used to determine the probability of the conditioned proposition(s). Probability is a relation between

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<sup>14</sup> More precisely, an adaptation of the Chain ladder method and its time series formulation in Buchwalder et al. Reference: Buchwalder M., Bühlmann H., Merz M. and M. V. Wüthrich: The Mean Square Error of Prediction in the Chain Ladder Reserving model (Mack and Murphy Revisited), ASTIN Bulletin 2006, no. 2, p. 521-542.

For information, the following reference may be useful (it has not been used by the participants): Buchwalder, M., Bühlmann, H., Merz, M. and M. V. Wüthrich: The Mean Square Error of Prediction in the Chain Ladder Reserving model (Mack and Murphy Revisited), ASTIN Bulletin 2006, no. 2, p. 521-542.

<sup>15</sup> Michael Merz, Mario V. Wüthrich (2006) "Prediction error of the expected claims development – result in the chain ladder method", published in Bulletin of the Swiss Association of Actuaries, 2007, no. 1, p. 117-137

<sup>16</sup> Mario V. Wüthrich, Michael Merz and Natalia Lysenko (2007) "Uncertainty in the claims development result in the chain ladder method"

conditioned hypothesis and conditioning information - it is meaningless to talk about THE probability of a hypothesis without also giving the evidence on which that probability value is based.

*Bayes' Theorem is a simple mathematical formula of conditional probabilities used in empirical learning.*

Bayes' Theorem uses conditional probabilities to reflect a degree of learning. It is central to model empirical learning both:

- because it simplifies the calculation of conditional probabilities and,
- because it clarifies significant features of the subjectivist position.

*Integration of new information in estimations*

Learning is a process of *belief revision* in which a "prior" subjective probability P is replaced by a "posterior" probability Q that incorporates newly acquired information.

This process proceeds in two stages:

- First, some of the subject's probabilities are *directly altered* by experience, intuition, memory, or some other *non-inferential* learning process.
- Second, the subject "updates" the rest of his/her opinions to bring them into line with his/her newly acquired knowledge.

## B.2/ Implementation of this concept in the reserving assessments

Participants have used an article written by David P.M. Scollnik. This article<sup>17</sup> is available at the following address: <http://math.ucalgary.ca/~scollnik/balducci/Papers/resnaaj.pdf><sup>18</sup>

This paper examines the use of Bayesian models for loss reserving inspired by a consideration of some of the methods and techniques which appear in the traditional chain ladder literature. As regards the current AISAM-ACME study, the most important added value of this approach is the possibility it provides to predict the reserve risk over a one year horizon. The implementation of these models is possible by using Markov chain Monte Carlo (MCMC) techniques and by using a specialized software program for MCMC simulation (WinBUGS). The code used to develop the methodology with WinBUGS is disclosed in this article. Nevertheless additional computations are necessary to achieve an assessment of the reserve risk over a one year horizon.

The WinBUGS software is free and available at the following address: <http://www.mrc-bsu.cam.ac.uk/bugs/winbugs/contents.shtml>

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<sup>17</sup> David P.M. Scollnik (2002) "Bayesian Reserving Models Inspired by Chain Ladder Methods and Implemented using Winbugs"

<sup>18</sup> This article is also available at the following address: [http://www.soa.org/library/research/actuarial-research-clearing-house/2004/october/arch04v38n2\\_3.pdf](http://www.soa.org/library/research/actuarial-research-clearing-house/2004/october/arch04v38n2_3.pdf)

For information the following article may provide additional information (it has not been used by participants): [http://www.soa.org/library/journals/north-american-actuarial-journal/2001/april/naaj0104\\_7.pdf](http://www.soa.org/library/journals/north-american-actuarial-journal/2001/april/naaj0104_7.pdf)

Remark: Mario V. Wüthrich has written an article<sup>19</sup> on the Bayesian methodologies with a reference to Scollnik's article: see the article at the following address: [http://www.math.ethz.ch/~wueth/Papers/BayesRes\\_V3.pdf](http://www.math.ethz.ch/~wueth/Papers/BayesRes_V3.pdf)

### B.3/ Adaptation of the existing methodologies to the one year horizon

To adapt the Bayesian methodology to the one year horizon the following procedure has been used:

In line with paragraph "4.1.2.2 Consequences of this framework on the reserve risk assessment", page 12, let us consider that:

- The aim is to assess the reserve risk at 31.12.N over a one year horizon,
- This reserve risk is related to the reserves at 31.12.N (Noted  $R_n$ ). These reserves are estimated as a best estimate (the risk margin is calculated as a second step),
- The cumulated payments until 31.12.N are  $P_n$  and,
- The ultimate cost is noted  $S_n$  ( $S_n = R_n + P_n$ )

The methodology is developed on the total costs (i.e. payments and reserves). The procedure consists of 5 steps:

- 1) We calculate the chain ladder development factors and the ultimate cost ( $S_n$ ) with a deterministic methodology. We therefore make an assessment of the ultimate costs (and therefore of the reserves) with information available at 31.12.N.
- 2) We calculate the stochastic diagonal of costs at N+1 by using the Bayesian model on the development factors. We introduce the random information during N+1 (the shock period).
- 3) The "shock period" N+1 enables us to make a new estimation of the ultimate costs (noted  $S_{n+1}$ ) with available information at 31.12.N+1 (the effect period)
- 4) Steps 2 and 3 can be run many times. Each run enables us to build the following variable:
 
$$\text{Run-off revision} = \frac{S_{n+1} - S_n}{R_n}$$
 This variable captures the revision of the ultimate costs between 31.12.N and 31.12.N+1 regarding the additional information arising during N+1.
- 5) The volatility of the variable Run-off revision provides the volatility of the reserve risk over a one year horizon.

### C/ The Bootstrap with Mack method

Bootstrapping methods do not allow for the interest variable to vary in the negative over time, a pattern restricting the use of this method to cumulative payments and

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<sup>19</sup> Mario V. Wüthrich (2007), "Using a Bayesian approach for claims reserving"

excluding any application with regard to the total charge. An alternative method to the classical Bootstrap has been developed to address this issue.

This method relies on several articles from Mack, England and Verrall:

- Mack (1993). 'Distribution-free calculation of the standard error of chain ladder estimates'. ASTIN Bulletin, Vol 23, No 2, 1993.
- England, Verrall (1998), 'Standard errors of prediction in claims reserving: a comparison of methods workshop', general insurance convention and ASTIN Colloquium, Glasgow, Scotland, 7-10 October 1998
- Mack (1999). 'The standard error of chain ladder reserve estimates: recursive calculations and inclusion of a tail factor'. ASTIN Bulletin, Vol 29, No 2, 1999, pp 361-366.
- England, Verrall (2002), 'Stochastic Claims reserving in general insurance', presented to the institute of actuaries, 28 January 2002.
- England, P. D. (2001). Addendum to 'Analytic and bootstrap estimates of prediction errors in claims reserving'. Actuarial Research Paper No. 138, Department of Actuarial Science and Statistics, City University, London, EC1V 0HB.

The prediction error can be split into the sum of the estimation error and the process variance.

#### ü Preliminary step: Calculation of the development factors' residuals

Let's consider the accident year  $i$ ,  $i \in [1, n]$  and the development year  $j$ ,  $j \in [1, n]$ .

The participant first assumes that the development factors  $f_{ij}$  are independent.

The participant estimates the development factors by applying the Chain Ladder method to the cumulated claim costs  $C_{ij}$ . The participant calculates the corresponding Mack standard error  $\hat{S}_j$ .

Applying the formula below, the participant calculates the development factors' residuals:

$$r_{ij} = \frac{f_{ij} - \hat{f}_j}{\sqrt{\frac{\hat{S}_j^2}{C_{ij}^{as\ if}}}} = \frac{\sqrt{C_{ij}^{as\ if}} * (f_{ij} - \hat{f}_j)}{\hat{S}_j}$$

with  $C_{ij}^{as\ if}$  the theoretical cumulated claims costs obtained by successively applying the development factors  $f_{ij}$  (actually we apply  $1/f_{ij}$ ) to the last diagonal of cumulated claims costs.

Estimation of the fitted residuals:

$$r_{ij}^a = \sqrt{\frac{\frac{n(n-1)}{2}}{\frac{n(n-1)}{2} - n}} \times r_{ij}$$

with  $n$  the size of the initial triangle,  $n(n-1)/2$  the number of residuals and  $n$  the number of estimated parameters.

Fitting the residuals is important to take into account the degrees of freedom in the calculation of the estimation error.

### ü Step 1 : Calculation of the estimation error

The participant applies the Bootstrap method to the triangle of fitted residuals: the participant calculates NS residuals triangles by random sampling with replacement

of the residuals. Let's consider  $r_{ij}^{a(k)}$  the residual corresponding to the accident year  $i$ , the development year  $j$  and the  $k^{\text{th}}$  simulation ( $k=1, \dots, \text{NS}$ ).

The participant rebuilds the NS development factors triangles from each simulated residuals triangle:

$$f'_{ij}{}^{(k)} = \hat{f}_j + \frac{r_{ij}^{a(k)} \hat{S}_j}{\sqrt{C_{ij}^{asif}}}, \text{ with } i+j \leq n+1.$$

From the last diagonal, the participant calculates the cumulated claim costs from the new development factors matrix:

$$C'_{ij}{}^{(k)} = \frac{C'_{i,j+1}}{f'_{ij}{}^{(k)}} \text{ with } C'_{i,j} = C_{i,j} \text{ for } i+j=n+1$$

The participant calculates the corresponding triangles of cumulated claims by applying the Chain Ladder method. The participant obtains the new  $\hat{f}'_j$ .

The estimation error is given by the standard error of the estimated reserves. As the residuals have been fitted, the estimation error includes the degrees of freedom.

### ü Step 2 : Calculation of the process error

At the  $k^{\text{th}}$  simulation the participant obtains a second set of simulations for the lower triangle with replacement of the residuals.

Let's consider  $C_{i,j}^{\text{proj}(k)}$  (for  $i+j > n+1$ ) the cumulated claims given by applying the development factors  $\hat{f}'^{(k)}_j$  to the upper triangle  $C'_{i,j}{}^{(k)}$ .

Let's consider  $f''_{ij}{}^{(k)}$  the development factors residuals for the accident year  $i$  and the development year  $j$  with  $i+j > n+1$ .

The following formula gives us the lower part of the projected triangle using residuals obtained from the 2<sup>nd</sup> set of simulations:

$$f''_{ij}{}^{(k)} = \hat{f}'_j{}^{(k)} + \frac{r''_{ij}{}^{a(k)} \hat{S}_j}{\sqrt{C''_{ij}{}^{proj(k)}}}$$

Then :

$$C''_{ij}{}^{(k)} = C'_{i,j-1}{}^{(k)} * f''_{ij}{}^{(k)} \quad \text{with } C''_{i,j} = C_{i,j} \text{ for } i+j=n+1$$

The reserves process error is given by the standard error of the NS simulations of the reserves.



## Appendix 2: Description of the models used by each participant.

Each participant who provides a stochastic assessment of the volatility of his/her company's provisions has been asked to detail the model(s), according to the following structure:

### Introduction

- *experience in modelling reserve risk,*
- *use in the day-to-day operations of the company,*

### A) Data

A.1) Data history

A.2) Specific treatment of data

### B) Data processing in prospective mode

B.1) Reserves modelling for the full run-off approach

B.2) Reserves modelling over the one year horizon

### C) Improvements

# Participant n° 1

## Introduction

This participant has been developing stochastic assessments of reserve risks for 3 years. The stochastic assessment of reserves is currently used to:

- Control the level of prudence of the technical provisions,
- Improve the asset/liabilities management.

## A) Data

### A.1) Data history

The valuation of the reserve risk relies on a claim-by-claim and a development year data history covering 15 years (compared with the average duration of liabilities which is 5). The data used are gross of reinsurance, so the net reserve is calculated with the relation net to gross.

### A.2) Specific treatment of data

The historical triangles are complete and the data have not been modified. In particular, the historical amounts of claims have not been deflated. The participant has found neither a relevant retrospective deflator nor a relevant prospective inflator. The participant has therefore decided to replicate the past level of inflation in the future, all the more so given that the line of business (workers' compensation) underwritten is not very sensitive to inflation changes over a short period of time.

## B) Data processing in prospective mode

### B.1) Reserves modelling for the full run-off approach

The Mack model is used to assess the volatility of the reserves over the whole run-off. The VaR are deduced from a lognormal fitting.

### B.2) Reserves modelling over the one year horizon

The participant calculated the one year approach by the methodology proposed by Merz and Wüthrich. The VaR are deduced from a lognormal fitting

## C) Improvements

As regards this study, the participant has identified two ways to improve his/her contribution:

- An assessment of the volatility net of reinsurance would avoid the overestimation of the volatilities
- Alternative methodologies have not yet been tested.

## Participant n°2

### Introduction

This participant has been developing stochastic assessments of reserve risks for 2/3 years. The stochastic assessment of reserves has been used in their replies to the 3 Quantitative Impact Studies carried out by the CEIOPS. This participant is used to working with deterministic methodology.

### A) Data

#### A.1) Data history

The valuation of the reserve risk relies on a claim-by-claim (occurrence basis) and a development year data history covering 17 years to 23 years (compared with the average duration of liabilities which is respectively about 4 and 5 years). The data used are gross of reinsurance so the assessments provide an over calibration of the required results.

#### A.2) Specific treatment of data

The historical triangles are complete (except for the year 1983) and the data have not been modified. The historical amounts of claims have been deflated (with a public index which properly reflects the inflation of the cost of claims) and then inflated with stochastic techniques (see the paragraph on the treatment of the inflation), except for the adaptation of the Mack method (past volatility implicitly replicated in the future).

### B) Data processing in prospective mode

The choice of the methodology relies on two drivers:

- The first driver is that the participant prefers working on the total charge of claims (payments + reserves) rather than incremental payments which are not reliable (i.e. not stable enough) for the most recent underwriting years regarding the long duration of the liabilities.
- The second driver is a consequence of the first one. Since the participant works on the total charge of claims, the models must be compatible with negative variations of the development factors.

#### B.1) Reserves modelling for the full run-off approach

##### B1.1) Bootstrap with Mack

Bootstrapping methods do not allow for the interest variable to vary in the negative over time, a pattern restricting the use of this method to cumulative payments and excluding any application with regard to the total charge. The participant therefore developed an alternative method to the classical Bootstrap. The participant used both Mack and Bootstrap theory to be able to work on total claim costs and deal with negative incremental data. The methodology is described in the scientific references.

## B1.2) Bayesian method

To calculate stochastic reserves, the participant uses a model based on the estimation of the development factors according to the Bayesian methodology. This model is described in the scientific references.

### Inflation for the full run-off approach:

Concerning the treatment of the inflation for the Bootstrap with Mack method: the participant considers three cases of future inflation selected randomly for each simulation (1st case: the future inflation is constant at 3% with a probability of 50%, 2nd case: inflation at 1.5% with a probability of 25%, 3rd case: inflation at 4.5% with a probability of 25%).

For the Bayesian method, the participant models the future inflation using a lognormal distribution with a mean at 3% and a volatility at 50%.

Note: for 2007, the median inflation is 4%, the other two cases being 2% or 6%.

## B.2) Reserves modelling over the one year horizon

Three methodologies have been tested:

- a mix of Mack model and Bootstrapping techniques,
- the Bayesian methodology,
- An adaptation of the Mack model (cf. Michaël Merz and Mario V. Wüthrich model).

### B2.1) Adjustment of the Bootstrap with Mack method to measure the one-year volatility

The one-year volatility is defined as the standard error of the run-off result variable. This variable measures the difference between the ultimate cost N+1 and N over the Reserves N (reserve for reported and unreported claims, calculated as a best estimate, BE):

The one-year volatility is defined as:

$$S \left( \frac{\text{Ultimate Cost } N+1 \text{ BE} - \text{Ultimate Cost } N \text{ BE}}{\text{Reserves } N \text{ BE}} \right)$$

The participant calculates the Reserves at 31.12.N and the best estimate of the ultimate cost with the available information at 31.12.N (called cost N) in a determinist way using the Chain Ladder method.

The ultimate cost N+1 is the ultimate cost calculated with the new and uncertain information over the next year (N+1). The diagonal N+1, which corresponds to the payments during the year N+1 and the claim by claim outstanding at the end of N+1, has to be calculated in a stochastic way.

The participant applies the Bootstrap with Mack method and gets 2 sets of simulations to identify the estimation error and the process error. Using the upper triangle made of historical data (to get the estimation error) and the diagonal N+1, the participant applies Chain Ladder in a determinist way to get the new triangle (size: N+1/N+1).

#### B2.2) Adjustment of the Bayesian method to measure the one-year volatility

The participant applied the Bayesian method detailed in the scientific references to estimate the run-off result variable defined as:

$$\text{Run-off result} = \frac{\text{Ultimate cost N + 1} - \text{Ultimate cost N}}{\text{Reserves BE N}}$$

The one-year volatility is given by the standard error of the run-off result variable.

#### B2.3) Adaptation of the Mack model

The participant has tested the methodology proposed by Merz and Wüthrich. The VaR are deduced from a lognormal fitting. This approach enables a split between the process error and the estimation error to be made easily.

##### Inflation for the one year horizon:

For the Bootstrap with Mack methodology, the participant applies stochastic inflation only for the estimation of the diagonal N+1. To simulate the inflation from 2006 to 2007, the participant considers three different cases (2% with a probability of 25%, 4% with a probability of 50% or 6% with a probability of 25%). For each simulation, one out of the three values is selected randomly.

For the Bayesian method, the participant models the future inflation using a lognormal distribution for 2007 with a mean at 4% and a volatility at 50%.

Then the participant assumes a 3% inflation per year to estimate the lower triangle in a determinist way.

For the adaptation of the Mack model (cf. Michaël Merz and Mario V. Wüthrich model), the historical amounts of claims have not been deflated. Indeed the participant has not implemented a specific inflation model within this model.

#### C) Improvements

Better modelling of the inflation (stochastic) and of the discount (or difference inflation-discount): the participant currently considers the free-rate curve used for the QIS3. The results are given for non-discounted reserves.

Finally the participant has chosen the Bootstrap with Mack method. The Bayesian Chain Ladder method does not seem to take sufficiently into account the estimation error. This method has to be studied in more detail.

**D) Results**

The participant obtains the following results for the prediction error (volatility).

	Method 1 Bootstrap with Mack			Method 2 Bayesian model			Method 3 Retrospective calculation based on historical datas	Method 4 Mack method (adapted by Merz- Wüthrich for 1 year horizon)			Method 5 Retrospective method based on accounting datas
	one year	Full run-off	Variation (%)	one year	Full run-off	Variation (%)	one year	one year	Full run-off	Variation (%)	one year
prediction error	7,6% (GL2) to 7,7% (GL1)	10,3% (GL1) to 11,2% (GL2)	-25% to -32%	5,5% (GL1) to 5,6% (GL2)	9,5% (GL2) to 9,6% (GL1)	-41% to -43%	8% (GL1)	3,6% (GL2) to 4,1% (GL1)	7,9% (GL2) to 8,3% (GL1)	-50% to -54%	2,9% (GL1)
estimation error								3% (GL2) to 3,2% (GL1)	6,6% (GL2) to 6,8% (GL1)	-53% to -55%	
process error								1,9% (GL2) to 2,5% (GL1)	4,4% (GL2) to 4,8% (GL1)	-48% to -57%	

## Participant n° 3

### Introduction

This participant has been developing stochastic assessments of reserve risks for 1-2 years. The stochastic assessment of reserves has been used in their replies to the Quantitative impact studies carried out by CEIOPS. For the day-to-day operations of the company the participant is used to working with deterministic methodology.

### A) Data

#### A.1) Data history

The valuation of the reserve risk relies on a claim-by-claim and a development year data history covering 23 years. The duration of the liabilities is around 7 years.

#### A.2) Specific treatment of data

The historical triangles of payments are complete since 1983 and the data have not been modified. The historical amounts of claims have not been deflated. The data used are gross of reinsurance. There is no specific treatment of large claims

### B) Data processing in prospective mode

For the full run-off approach the participant applied the Mack model<sup>20</sup>. Over the one year horizon, the participant used the Michaël Merz and Mario V.Wüthrich model.

#### B.1) Reserves modelling for the full run-off approach

The Mack model:

- Estimation of the average ultimate cost of claims using the standard Chain Ladder method.
- Calculation of the standard error of the ultimate cost using the Mack method.
- The VaR are deduced from a lognormal fitting
- The full run off volatility is defined as standard error (reserve BE)/reserve BE

#### B.2) Reserves modelling over the one year horizon approach

An adaptation of the Mack model (cf. Michaël Merz and Mario V.Wüthrich model) has been used on the same data as for the run-off approach.

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<sup>20</sup> Reference: Measuring the variability of the chain ladder reserve estimates by T.Mack

**C) Improvements**

- The use of an alternative method such as the Bayesian methodology would be useful
- The traditional bootstrapping methods are tested but they need to be improved.
- An assessment of the volatility net of reinsurance would avoid the overestimation of the volatilities



## Participant n°5

### Introduction

- This participant has been developing stochastic models of reserves risk for 4 years
- These projections have been used more particularly to make decisions in
  - Tarification,
  - Asset/Liabilities management,
  - Solvency Capital required, rating of the company.

### A) Data

#### A.1) Data history

#### Concerning the general liabilities line of business:

- The participant has considered the cost of claims notified between 1980 and 2006. The participant only has the claim costs since calendar year 1990, the data being unavailable for the previous accounting years.
- The duration of liabilities is 11 years.

#### Concerning the workers' compensation line of business:

- The valuation of the reserve risk relies on a claim-by-claim and a development year data history covering 10 years (compared with the average duration of liabilities which is 2.9).
- The data used are gross of reinsurance so the assessments provide an over calibration of the required results.

#### A.2) Specific treatment of data

#### Concerning the general liabilities line of business:

- The participant has completed the lower part of the triangle until the 20th development year on a current money historical data basis. Historical amounts have not been deflated and retrospective inflation is repeated in the future. After 20 years, the participant has made a claim-by-claim projection of future cash-flows.
- The data used are gross of reinsurance.

#### Concerning the workers' compensation line of business:

The historical triangles are complete and the data have not been modified. In particular, the historical amounts of claims have not been deflated. The participant has found neither a relevant retrospective deflator nor a relevant prospective inflator. The participant has therefore decided to replicate the past level of inflation in the future.

### B) Data processing in prospective mode

Concerning the general liabilities line of business:

The participant has implemented different models to estimate the reserves volatility. For the full run-off approach the participant has applied both the Mack model and a Bayesian approach<sup>(1)</sup>. Over the one year horizon, the participant has used both the Michaël Merz and Mario V. Wüthrich model<sup>(2)</sup> and a Bayesian approach. This participant has adapted these models to make them consistent with his/her company's own risk.

Concerning the workers' compensation line of business:

The Mack model is used to assess the volatility of the reserves over the whole run-off. The VaR are deduced from a lognormal fitting. The participant calculated the one year approach using the methodology proposed by Merz and Wüthrich. The VaR are deduced from a lognormal fitting

The following developments concern the general liabilities line of business only.

#### B.1) Reserves modelling for the full run-off approach

##### B.1.1) The Mack model

The Mack model consists of 4 steps:

- Estimation of the average ultimate cost of claims notified between 1987 and 2006 using the standard Chain Ladder method.
- Calculation of the standard error of the ultimate cost using the Mack method.
- Using the parameters obtained in step 1 and 2, simulation of the ultimate costs which are assumed to be normal correlated random variables.
- Modelling of the payments frequency with a Beta distribution to get the payments projection to 20 years.

The participant has added the payments simulated beyond 20 years (the projections are done claim-by-claim) and has deduced the reserves Best Estimate.

The full run-off volatility is defined as: 
$$\frac{\text{predictive error given by Wüthrich Merz}}{E[\text{Reserves BE}]}$$

##### B.1.2) The Bayesian model

The Bayesian model for the full run-off consists of 3 steps:

- Step 1: Application of the Chain Ladder method to the historical data to get prior parameters used in step 2.
- Step 2: Application of the Bayesian methodology described in the scientific references to estimate the ultimate claim cost. On a full run-off approach, the only difference consists in estimating all the diagonals in a stochastic way and not only the diagonal N+1.
- Step 3: Modelling of the payments frequency with a Beta distribution to get the payments' projection to 20 years.

The participant has added the payments simulated beyond 20 years (the projections are done claim-by-claim) and has deduced the reserves Best Estimate.

Unlike the Wüthrich and Merz model, the Bayesian model does not require any fitting to any distribution on the ultimate cost.

The full run-off volatility is defined as: 
$$\frac{S(\text{Reserves BE})}{E[\text{Reserves BE}]}$$

## B.2) Reserves modelling over the one year horizon

### B.2.1) The Michaël Merz and Mario V.Wüthrich model

The method is the same as the one described for the full run-off except that the parameters in step 2 have been estimated using the Michaël Merz and Mario V.Wüthrich method (c.f. scientific references). This model has been implemented using both cumulated payments and claim costs and gives close results.

The one-year volatility is defined as: 
$$\frac{\text{predictive error given by Wüthrich Merz}}{E[\text{Reserves BE}]}$$

### B.2.2) The Bayesian model

The methodology applied for the one-year volatility is exactly the same as the one for the full run-off, except that only the diagonal N+1 is estimated in a stochastic way.

The one-year volatility is defined as:

$$S\left(\frac{\text{Ultimate Cost } N+1 \text{ BE} - \text{Ultimate Cost } N \text{ BE}}{\text{Reserves } N \text{ BE}}\right)$$

## C) Improvements

An assessment of the volatility net of reinsurance would avoid the overestimation of the volatilities (both for general liabilities and workers' compensation).

## D) Results

The participant obtains the following results for the prediction error (volatility).

**General liabilities**

Method 1 : Michael Merz, Mario V. Wüthrich		Method 2 : Bayesian model	
1 year	Full run_off	1 year	Full run_off
5,4%	10,6%	5,9%	15,6%

**Worker's compensation**

Method 1 : Michael Merz, Mario V. Wüthrich		Method 2 : Bayesian model	
1 year	Full run_off	1 year	Full run_off
6,4%	8,8%	6,3%	9,5%

## Participant n°9

### Introduction

This participant has been developing stochastic assessments of reserve risks for 15 years. The reserve risk is a sub module of the internal model which is not totally built yet. The stochastic assessment of reserves is currently used to:

- Control the level of prudence of the technical provisions,
- Check the adequacy of tariffs,
- Run an Asset/Liability Management system.

### A) Data

#### A.1) Data history

The valuation of the reserve risk relies on a claim-by-claim and a development year data history covering 20 years (compared with the average duration of liabilities which is 6) so as to have at least 5 observations for the latest Chain-Ladder development factors. The data used are gross of reinsurance, so the assessments provide an over calibration of the required results.

#### A.2) Specific treatment of data

The historical triangles are complete and the data have not been modified. In particular, the historical amounts of claims have not been deflated. The participant has found neither a relevant retrospective deflator nor a relevant prospective inflator. The participant has therefore decided to replicate the past level of inflation in the future, all the more so given that the line of business (general liability) underwritten has faced a strong inflation trend in the past and significant shocks in the patterns of payments because of adverse court cases. To that extent the use of historical data without changes appears to be cautious.

### B) Data processing in prospective mode

#### B.1) Reserves modelling for the full run-off approach

The Mack model is used to assess the volatility of the reserves over the whole run-off. The VaR are deduced from a lognormal fitting. Through a strong “use experience”, this model has proven its efficiency and reliability for the business of this participant.

#### B.2) Reserves modelling over the one year horizon

Two methodologies have been developed: an adaptation of the Mack model and an adaptation of the Bayesian methodology

An adaptation of the Mack methodology<sup>21</sup> has been used on the same set of data as for the whole run-off approach.

### C) Improvements

An assessment of the volatility net of reinsurance would avoid the overestimation of the volatilities

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<sup>21</sup> Wüthrich, Merz

## Participant n° 10

### Introduction

This participant has been developing stochastic assessments of reserve risks for 15 years. The stochastic assessment of reserves is currently used to control the level of prudence of the technical provisions

### A) Data

#### A.1) Data history

The valuation of the reserve risk relies on a claim-by-claim and a development year data history covering 15 years (compared with the average duration of liabilities which is 5). The data used are gross of reinsurance, so the net reserve is calculated with the relation net to gross. This corresponds to the suggestion of the financial supervisory agency.

#### A.2) Specific treatment of data

Considering the participant's high non-proportional reinsurance protection and even though the gross of reinsurance triangle was used, the participant eliminated one link ratio in his/her development triangle so as to smooth an exceptional claim. If the net of reinsurance triangle had been used, this re-treatment would not have been necessary.

In addition, the historical amounts of claims have not been deflated. The participant has found neither a relevant retrospective deflator nor a relevant prospective inflator. To that extent the use of historical data without changes appears to be a good solution.

### B) Data processing in prospective mode

#### B.1) Reserves modelling for the full run-off approach

The Mack model is used to assess the volatility of the reserves over the whole run-off. The VaR are deduced from a lognormal fitting.

#### B.2) Reserves modelling over the one year horizon

The participant calculated the one year approach using the methodology proposed by Merz and Wüthrich. The VaR are deduced from a lognormal fitting

### C) Improvements

As regards this study, the participant has identified two ways to improve his/her contribution:

- An assessment of the volatility net of reinsurance would avoid the overestimation of the volatilities
- Bootstrapping and Bayesian methodologies have not been tested yet.

